A NEW FORMULATION FOR YIELD OPTIMIZATION

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ABSTRACT

A new and elegant formulation for yield optimization is presented with applications to microwave circuits. It extends the conventional discrete Monte Carlo estimate of yield to a continuous *yield probability function* suitable for gradient-based optimization. The merit of our new method is demonstrated by yield optimization of an amplifier and a nonlinear FET doubler.

INTRODUCTION

Yield optimization must be included as an integral part of the design process in order to improve first-time design success rate and to reduce manufacturing cost. Many different approaches to design centering/yield optimization have been explored. Methods which attempt to optimize the theoretical yield as an integral function are usually too complicated for practical implementation in circuit CAD programs. Typically, circuit CAD programs estimate yield by Monte Carlo simulation.

The yield estimated by Monte Carlo simulation is a discrete function and cannot be directly optimized by gradient-based methods. The new formulation introduced in this paper extends the classical Monte Carlo concept by replacing the discrete acceptance index (outcome passed or failed) with a continuous *yield probability function* suitable for gradient-based optimization.

We demonstrate the merit of our new formulation by the yield optimization of a small-signal amplifier and a nonlinear FET frequency doubler. We compare the effectiveness and efficiency of the new formulation with the one-sided ℓ_1 centering approach by Bandler *et al.* [1-3].

The new yield probability formulation is implemented in the general-purpose microwave circuit CAD/CAE system OSA90/hope™ [4]. This system was used to produce all the numerical results contained in this paper.

This work was supported in part by Optimization Systems Associates Inc. and in part by the Natural Sciences and Engineering Research Council of Canada under Grants OGP0007239, OGP0042444 and STR0040923.

The authors are also with the Simulation Optimization Systems Research Laboratory and the Department of Electrical and Computer Engineering, McMaster University, Hamilton, Canada L8S 4L7.

YIELD PROBABILITY FUNCTION

Let ϕ denote the vector of circuit parameters subject to tolerances and other statistical variations. We can define a generalized least *p*th error function $H(\phi)$ [1,5] such that

$$\begin{aligned} H(\phi) &\le 0 \quad \text{if all specifications are satisfied} \\ H(\phi) &> 0 \quad \text{if some specifications are violated} \end{aligned} \tag{1}$$

The theoretical yield can be represented by the probability

$$P(H(\phi) \le 0) \tag{2}$$

given the statistical distribution of ϕ.

In Monte Carlo simulation, a finite number of random outcomes are sampled from the distribution of ϕ, and the yield is estimated by the percentage of acceptable outcomes. If we define an acceptance index $I(\phi)$ such that

$$I(\phi) = \begin{cases} 1 & \text{if } H(\phi) \le 0 \\ 0 & \text{if } H(\phi) > 0 \end{cases} \tag{3}$$

then the Monte Carlo yield estimate can be expressed as

$$Y = \sum I(\phi^k) \;/\; N \tag{4}$$

where the summation is over k = 1, 2, ... N, and N is the total number of outcomes. The estimate given by (4) is not suitable as an objective function for gradient-based optimization due to the discrete nature of $I(\phi)$ and its discontinuity at $H(\phi) = 0$.

Our motivation is to find a substitute for $I(\phi)$ that is more suitable for gradient-based optimization. We consider a neighborhood of the outcome ϕ^k, denoted by Ω^k, and represent the yield in Ω^k by the probability

$$P^k = P(H(\phi) \le 0 \mid \phi \in \Omega^k) \tag{5}$$

Then the overall yield can be estimated by

$$Y = \sum P^k \;/\; N \tag{6}$$

The classical Monte Carlo estimate (4) becomes a special case of (6) when Ω^k consists of a single outcome ϕ^k and consequently $P^k = I(\phi^k)$. If we extend Ω^k from a point to a *region* P^k becomes a continuous measure of the intersection of Ω^k and the acceptability region, and the value of P^k varies continuously between 0 and 1.

To evaluate P^k precisely as defined by (5), we will need to know the distribution of $H(\phi)$ in Ω^k. As an approximation,

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we may use the sample means of $H(\phi)$, denoted by $\overline{H}(\phi)$, to replace $H(\phi)$ in (5):

$$P^k \approx \overline{P}^k = P(\overline{H}(\phi) \le 0 \mid \phi \in \Omega^k) \quad (7)$$

According to the *central limit theorem* [6], the frequency distribution of $\overline{H}(\phi)$ can be assumed to be normal (Gaussian). Hence, $\overline{P}^k$ in (7) is the probability of a normal distribution which can be computed in a straightforward manner using, for instance, the Hastings formula [7]:

$$\overline{P}^k = \begin{cases} 1 - f & \text{if } \mu < 0 \\ f & \text{if } \mu \ge 0 \end{cases}$$

$$f = 0.5\,(a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4 + a_5 t^5)\, e^{-x^2} \quad (8)$$

$$x = \mu \,/\, (\sqrt{2}\,\sigma)$$

$$t = 1 \,/\, (1 + 0.3275911\,|x|)$$

where μ and σ are the mean value and standard deviation of $\overline{H}(\phi)$ in Ω^k, respectively, and $a_1 = 0.2548296$, $a_2 = -0.2844967$, $a_3 = 1.421214$, $a_4 = -1.453152$ and $a_5 = 1.061405$. Fig. 1 depicts $\overline{P}^k$ as a function of μ.

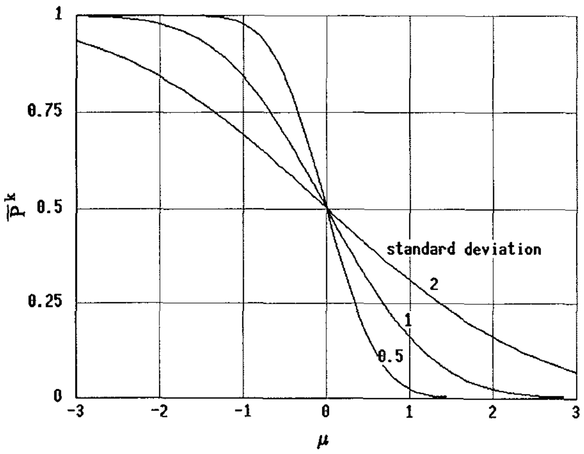

Fig. 1. Yield probability function $\overline{P}^k$ versus μ.

Since Ω^k is defined conceptually as a small neighborhood of ϕ^k, we use $H(\phi^k)$ as μ in Ω^k. When $H(\phi^k) >> 0$, ϕ^k is far away from the acceptable region, and therefore the yield probability $\overline{P}^k$ approaches 0. When $H(\phi^k) = 0$, ϕ^k is on the boundary of the acceptable region and $\overline{P}^k = 0.5$. As ϕ^k moves inside the acceptable region ($H(\phi^k) < 0$), $\overline{P}^k$ approaches 1. The standard deviation of $\overline{H}(\phi)$, σ, provides a measure of the circuit performance variation in Ω^k and affects $\overline{P}^k$ as a scaling factor.

Our new method is implemented in the general-purpose microwave CAD/CAE system OSA90/hope™ [4]. This system was used to produce the application results presented in this paper. In this implementation $H(\phi)$ is chosen as a generalized least squares function (i.e., $p = 2$). We define the objective function as

$$F = 1 - \sum \overline{P}^k \,/\, N \quad (9)$$

to transform the maximization of the estimated yield into the minimization of F. Here, N is the total number of outcomes involved in yield optimization and the summation is over $k = 1, 2, ..., N$. A quasi-Newton optimizer based on Powell's algorithm [8] is employed to minimize F.

SMALL-SIGNAL AMPLIFIER

We consider yield optimization of a single-stage small-signal 10MHz to 1GHz amplifier [9], as shown in Fig. 2. The design specifications are input/output return loss ≤ -8dB, and 9dB ≤ gain ≤ 10dB. The parameters R_1, L_1, L_2 and C_2 are considered as design variables.

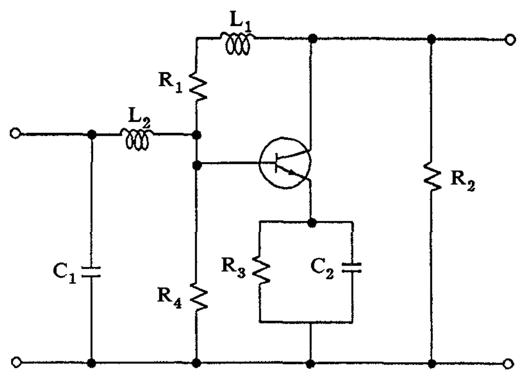

Fig. 2. Single-stage small-signal amplifier [9].

First, a nominal design is obtained by minimax optimization. At the nominal solution, tolerances were assumed for the parameters R_1, R_3, L_2 and C_1, as listed in Table I. The yield was estimated as 37.8% by Monte Carlo analysis with 500 random outcomes.

TABLE I

SMALL-SIGNAL AMPLIFIER PARAMETERS

Parameter	Nominal Design	After Yield Optimization	Tolerance* or Standard Deviation†
R_1 (Ω)	154.2	151.3	10%†
R_2 (Ω)	300.0	300.0	-
R_3 (Ω)	5.0	5.0	5%†
R_4 (Ω)	550.0	550.0	-
L_1 (nH)	23.39	25.34	-
L_2 (nH)	8.269	7.283	15%*
C_1 (pF)	4.0	4.0	5%†
C_2 (pF)	15.09	10.55	-

* Assumed tolerance of uniform distribution
† Assumed standard deviation of Gaussian distribution

After yield optimization using our new method, the yield was increased to 50%. 20 outcomes were used in the yield optimization (500 outcomes were used in the Monte Carlo analyses before and after optimization). The yield optimization took 30 seconds of CPU time on a Sun SPARCstation 1.

For comparison, we repeated the yield optimization using the one-sided ℓ_1 centering algorithm [1-3]. Using the same number of outcomes, the yield of the centered design was

42.6%, and then further increased to 45.6% after a restart of optimization (it took 33 seconds of CPU time including the restart). The problem is that the yield predicted by the one-sided ℓ_1 algorithm (based on 20 outcomes) is not close enough to the yield estimated by Monte Carlo analysis (based on 500 outcomes). When 100 outcomes were included in optimization, the yield was increased to 50% (total CPU time: 96 seconds). Here, the new method demonstrates a clear advantage: no restart of optimization is needed; the predicted yield is more precise and hence a better solution (higher yield) is achieved without increasing the number of outcomes in optimization.

The parameter values after yield optimization are listed in Table I. Fig. 3 shows the run charts of the amplifier gain at 1GHz before and after yield optimization.

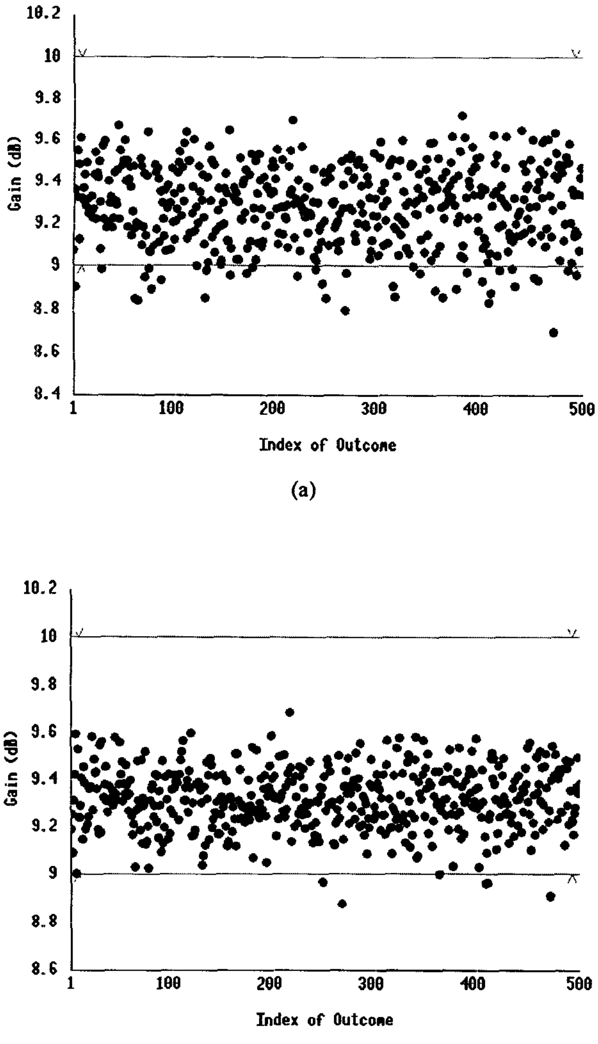

Fig. 3. Run charts of the small-signal amplifier gain at 1GHz obtained from Monte Carlo analysis: (a) before yield optimization, and (b) after yield optimization. The horizontal lines represent the specifications.

NONLINEAR FET DOUBLER

We now consider an application to nonlinear microwave circuits. Fig. 4 depicts a nonlinear FET frequency doubler [3]. It consists of a FET characterized by a large-signal statistical model and input/output matching networks. Details of the parameter values and the FET statistics are given in [3]. The input fundamental frequency is 5GHz. The specifications are conversion gain ≥ 2.5dB and spectral purity ≥ 19dB.

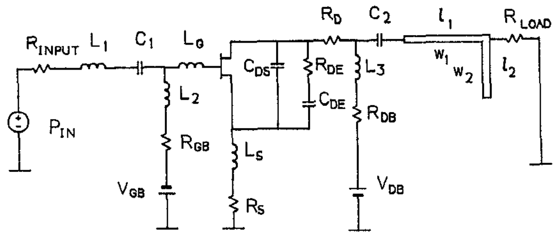

Fig. 4. Nonlinear FET frequency doubler.

The yield of the doubler was 31% for the nominal design (predicted by Monte Carlo analysis with 500 outcomes). The yield was improved to 74% after optimization using the new method. The optimizable variables included the lumped input matching network element L_1 and the microstrip lengths l_1 and l_2. Their values are listed in Table II. The number of outcomes involved in optimization was 50 and the CPU time consumed was 20 minutes. Fig. 5 shows the histograms of the doubler spectral purity before and after yield optimization.

TABLE II

OPTIMIZABLE VARIABLES OF THE DOUBLER

Optimization Variable	Before Yield Optimization	After Yield Optimization
L_1 (nH)	5.462	5.350
l_1 (mm)	1.483	1.655
l_2 (mm)	5.771	5.915

In comparison, the yield optimized by the one-sided ℓ_1 centering algorithm [1-3] with the same number of outcomes was 74.6% (19 CPU minutes). In this case, the solutions and efficiency of the two methods are very similar.

CONCLUSIONS

We have presented a new formulation for circuit yield optimization. We have introduced a yield probability function to replace the discrete and discontinuous acceptance index in the classical Monte Carlo analysis. This has led to a yield estimate suitable for gradient-based optimization.

The new formulation has been implemented in the microwave circuit CAD/CAE system OSA90/hope. We have demonstrated its effectiveness and efficiency through relevant circuit applications, and the results compare favorably with the well-established one-sided ℓ_1 centering algorithm.

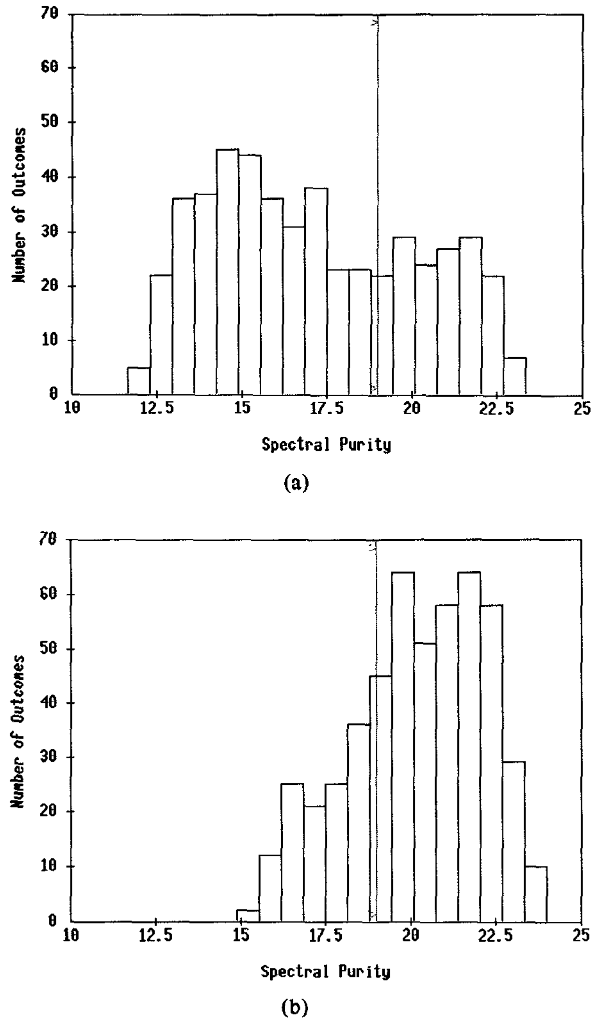

Fig. 5. Histograms of the doubler spectral purity: (a) before yield optimization, and (b) after yield optimization. The vertical line represents the specification.

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